4726 Mark Scheme June 2010

- 1 Derive/quote $g'(x) = p/(1+x^2)$ Attempt f'(x) as $a/(1+bx^2)$ Use $x = \frac{1}{2}$ to set up a solvable equation in p, leading to at least one solution Get $p = \frac{5}{4}$ only
- Reasonable attempt at e^{2x} (1+2x+2x²)
 Multiply out their expressions to get all terms up to x²
 Get 1+3x+4x²
 Use binomial, equate coefficients to get 2 solvable equations in a and n
 Reasonable attempt to eliminate a or n
 Get n=9, a=1/3 cwo

3 Quote/derive correct $dx=2dt/(1+t^2)$ Replace all x (not dx=dt) Get $2/(t-1)^2$ or equivalent Reasonable attempt to integrate their expression Use correct limits in their correct integral Clearly tidy to $\sqrt{3}+1$ from cwo

4 (i) Get a = -2 Get b = 6 Get c = 1

(ii) √6 √6

B1 M1 Allow any *a*, *b*=2 or 4

M1

A1 AEEF

M1 3 terms of the form $1+2x+ax^2$, $a\neq 0$

M1 (3 terms) x (minimum of 2 terms)

A1 cao

Reasonable attempt at binomial, each term

M1 involving a and n (an=3, $a^2n(n-1)/2=4$)

M1

A1 cao

SC Reasonable f '(x) and f "(x) using product rule (2 terms) M1
Use their expressions to find f '(0) and f "(0) M1
Get 1+3x+4x² cao A1

B1 M1 From their expressions A1

M1

A1 $\sqrt{1}$ Must involve $\sqrt{3}$

A1 A.G.

B1 May be quoted B1 May

B1 Correct shape in $-1 < x \le 3$ only (allow just top or bottom half)

B1 90^0 (at x=3) (must cross x-axis i.e. symmetry)

B1 Asymptote at x=-1 only (allow -1 seen)

B1 $\sqrt{\frac{b}{c}}$ Correct crossing points; $\pm \sqrt{\frac{b}{c}}$ from their b,c

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5 (i) Reasonable attempt at parts Get $e^x(1-2x)^n$ - $\int e^x .n(1-2x)^{n-1}2 dx$ Evidence of limits used in integrated part Tidy to A.G.	M1 Leading to second integral A1 Or $(1-2x)^{n+1}/(-2(n+1))e^x$ $-\int (1-2x)^{n+1}/(-2(n+1))e^x dx$ M1 Should show ± 1 A1 Allow $I_{n+1} = 2(n+1)I_n - 1$
(ii) Show any one of $I_3=6I_2-1$, $I_2=4I_1-1$, $I_1=2I_0-1$ Get $I_0(=e^{1/2}-1)$ or $I_1(=2e^{1/2}-3)$ Substitute their values back for their I_3 Get $48e^{1/2}-79$	B1 May be implied B1 M1 Not involving <i>n</i> A1
6 (i) Reasonable attempt to differentiate sinh $y = x$ to get dy/dx in terms of y Replace sinh y to A.G.	M1 Allow $\pm \cosh y dy/dx = 1$ A1 Clearly use $\cosh^2 - \sinh^2 = 1$ SC Attempt to diff. $y = \ln(x + \sqrt{x^2 + 1})$ using chain rule M1 Clearly tidy to A.G. A1
(ii) Reasonable attempt at chain rule Get $dy/dx = a \sinh(a\sinh^{-1}x)/\sqrt{(x^2+1)}$ Reasonable attempt at product/quotient Get d^2y/dx^2 correctly in some form Substitute in and clearly get A.G.	M1 To give a product A1 M1 Must involve sinh and cosh A1 $\sqrt{\text{From d}y/\text{d}x} = k \sinh(a\sinh^{-1}x)/\sqrt{(x^2+1)}$ A1 SC Write $\sqrt{(x^2+1)}\text{d}y/\text{d}x = k \sinh(a\sinh^{-1}x)$ or similar Derive the A.G.
7 (i) Get 5.242, 5.239, 5.237 Get 5.24	B1√ Any 3(minimum) correct from previous value B1 Allow one B1 for 5.24 seen if 2 d.p.used
(ii) Show reasonable staircase for any region Describe any one of the three cases Describe all three cases	B1 Drawn curve to line B1 B1
(iii) Reasonable attempt to use log/expo. rule Clearly get A.G.Attempt f'(x) and use at least once in correct N-R formulaGet answers that lead to 1.31	M1 A1 Minimum of 2 answers; allow truncation/rounding to at least 3 d.p.
(iv) Show f'(ln36) = 0 Explain why N-R would not work	B1 B1 Tangent parallel to Ox would not meet Ox again

or divide by 0 gives an error

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8 (i) Use correct definition of $\cosh x$ Attempt to cube their definition

involving e^x and e^{-x} (or e^{2x} and e^x)

M1 Must be 4 terms

Put their 4 terms into LHS and attempt
to simplify

Clearly get A.G.

M1 Must be 4 terms

M1

A1

SC Allow one B1 for correct derivation from cosh3x = cosh(2x+x)

(ii) Rewrite as $k \cosh 3x = 13$ M1
Use ln equivalent on 13/k M1 Allow $\pm \ln \text{ or } \ln(13/k \pm \sqrt{(13/k)^2 - 1})$ for their k or attempt to set up and solve quadratic via exponentials

Get $x = (\pm) \frac{1}{3} \ln 5$ A1 Replace in $\cosh x$ for u M1 Use $e^{a \ln b} = b^a$ at least once M1 Get $\frac{1}{2}(5^{\frac{1}{3}} + 5^{-\frac{1}{3}})$ A1

9 (i) Attempt integral as $k(2x+1)^{1.5}$ M1 Get 9 A1 cao

Attempt subtraction of areas M1 Their answer – triangle Get 3 A1 $\sqrt{}$ Their answer – 6 (>0)

(ii) Use $r^2 = x^2 + y^2$ and $x = r\cos\theta$, $y = r\sin\theta$ B1 Eliminate x and y to produce quadratic equation (=0) in r (or $\cos\theta$) M1 Solve their quadratic to get r in terms of θ (or vice versa) A1 $\sqrt{\frac{1}{2}}$

Clearly get A.G. A1 r>0 may be assumed

Clearly show $\theta_1(at B)=\tan^{-1}3/4$ and

 $\theta_2 (at A) = \pi$ B1

SC Eliminate y to get r in terms of x only M1 Get r = x + 1 A1 SC Start with $r=1/(1-\cos\theta)$ and derive cartesian

(iii) Use area = $\frac{1}{2}\int r^2 d\theta$ with correct r B1 cwo; ignore limits Rewrite as $k \csc^4(\frac{1}{2}\theta)$ M1 Not just quoted Equate to their part (i) and tidy Get 24 M1 To get $\int = \text{some constant}$ A1 A.G.